

数と式(展開)(解答)NO1

1. 次の式を x について降べきの順に整理せよ。

$$x + 2y^2 + 3x^2 - 5y + 3 + 2xy = 3x^2 + (2y + 1)x + (2y^2 - 5y + 3)$$

2. 次の式を展開せよ。

乗法公式

$$[6](a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \quad [7](a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$[8](a + b)(a^2 - ab + b^2) = a^3 + b^3 \quad [9](a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$(1) (2x + 3)^3 = (2x)^3 + 3 \cdot (2x)^2 \cdot 3 + 3 \cdot (2x) \cdot 3^2 + 3^3 = 8x^3 + 36x^2 + 54x + 27$$

$$(2) (3a - 2b)^3 = (3a)^3 - 3 \cdot (3a)^2 \cdot (2b) + 3 \cdot (3a) \cdot (2b)^2 - (2b)^3 = 27a^3 - 54a^2b + 36ab^2 - 8b^3$$

$$(3) (2a + b)(4a^2 - 2ab + b^2) = (2a + b)\{(2a)^2 - (2a) \cdot b + b^2\} = 8a^3 + b^3$$

$$(4) (3x - 4y)(9x^2 + 12xy + 16y^2) = (3x - 4y)\{(3x)^3 + (3x) \cdot (4y) + (4y)^2\} = 27x^3 - 64y^3$$

置き換えによる展開の工夫

3. 次の式を展開せよ。

$$(1) (a + b - 1)(a + b + 3)$$

$$a + b = A \text{ とおくと}$$

$$\begin{aligned} (a + b - 1)(a + b + 3) &= (A - 1)(A + 3) \\ &= A^2 + 2A - 3 = (a + b)^2 + 2(a + b) - 3 \\ &= a^2 + 2ab + b^2 + 2a + 2b - 3 \end{aligned}$$

$$(2) (a + b - c)(a - b + c)$$

$$b - c = A \text{ とおくと}$$

$$\begin{aligned} (a + b - c)(a - b + c) &= \{a + (b - c)\}\{a - (b - c)\} \\ &= (a + A)(a - A) = a^2 - A^2 = a^2 - (b - c)^2 \\ &= a^2 - (b^2 - 2bc + c^2) = a^2 - b^2 + 2bc - c^2 \end{aligned}$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

4. 次の式を展開せよ。

$$\begin{aligned} (1) (2x + y - z)^2 &= 4x^2 + y^2 + z^2 + 2 \cdot (2x) \cdot y + 2 \cdot y \cdot (-z) + 2 \cdot (-z) \cdot (2x) \\ &= 4x^2 + y^2 + z^2 + 4xy - 2yz - 4zx \end{aligned}$$

応用問題

$$(1) (x + 1)(x + 2)(x + 3)(x + 4)$$

$$= (x + 1)(x + 4) \times (x + 2)(x + 3)$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6)$$

$$x^2 + 5x = A \text{ とおくと、}$$

$$(x + 1)(x + 2)(x + 3)(x + 4) = (A + 4)(A + 6)$$

$$= A^2 + 10A + 24 = (x^2 + 5x)^2 + 10(x^2 + 5x) + 24$$

$$= x^4 + 10x^3 + 25x^2 + 10x^2 + 50x + 24$$

$$= x^4 + 10x^3 + 35x^2 + 50x + 24$$

$$(2) (x + 1)(x + 2)(x + 3)(x + 6)$$

$$= (x + 1)(x + 6) \times (x + 2)(x + 3)$$

$$= (x^2 + 7x + 6)(x^2 + 5x + 6)$$

$$x^2 + 6 = A \text{ とおくと、}$$

$$(x + 1)(x + 2)(x + 3)(x + 6) = (A + 7)(A + 5)$$

$$= A^2 + 12Ax + 35x^2$$

$$= (x^2 + 6)^2 + 12(x^2 + 6)x + 35x^2$$

$$= x^4 + 12x^3 + 36 + 12x^3 + 72x + 35x^2$$

$$= x^4 + 12x^3 + 47x^2 + 72x + 36$$

数と式(展開)(解答)NO2

$$\begin{aligned}
 (3) & (a-b)(a+b)(a^4+a^2b^2+b^4) \\
 &= (a^2-b^2)(a^4+a^2b^2+b^4) \\
 &= (a^2)^3 - (b^2)^3 \\
 &= a^6 - b^6
 \end{aligned}$$

$$\begin{aligned}
 (4) & (a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2) \\
 &= (a-b)(a^2+ab+b^2) \times (a+b)(a^2-ab+b^2) \\
 &= (a^3-b^3)(a^3+b^3) \\
 &= a^6 - b^6
 \end{aligned}$$

$$\begin{aligned}
 (5) & (2x+3)^2(2x-3)^2 \\
 &= \{(2x+3)(2x-3)\}^2 \\
 &= (4x^2-9)^2 \\
 &= 16x^4 - 72x^2 + 81
 \end{aligned}$$

$$\begin{aligned}
 (6) & (x-2)^3(x-2)^3 \\
 &= \{(x-2)(x+2)\}^3 \\
 &= (x^2-4)^3 \\
 &= (x^2)^3 - 3(x^2)^2 \cdot 4 + 3x^2 \cdot 4^2 - 4^3 \\
 &= x^6 - 12x^4 + 48x^2 - 64
 \end{aligned}$$

$$\begin{aligned}
 (7) & (a-b)(a+b)(a^2+b^2)(a^4+b^4) \\
 &= (a^2-b^2)(a^2+b^2)(a^4+b^4) \\
 &= (a^4-b^4)(a^4+b^4) \\
 &= a^8 - b^8
 \end{aligned}$$

$$\begin{aligned}
 (8) & (a-b)^3(a+b)^3(a^2+b^2)^3 \\
 &= \{(a-b)(a+b)(a^2+b^2)\}^3 \\
 &= \{(a^2-b^2)(a^2+b^2)\}^3 = (a^4-b^4)^3 \\
 &= (a^4)^3 - 3(a^4)^2(b^4) + 3a^4(b^4)^2 - (b^4)^3 \\
 &= a^{12} - 3a^8b^4 + 3a^4b^8 - b^{12}
 \end{aligned}$$

5. 次の式を展開せよ。

$$\begin{aligned}
 (1) & (3a-2b)^3 \\
 &= 27a^3 - 54a^2b + 36ab^2 - 8b^3
 \end{aligned}$$

$$\begin{aligned}
 (2) & (2a-3b-c)^2 \\
 &= 4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca
 \end{aligned}$$

$$\begin{aligned}
 (3) & (x+3y-2z)(x-3y+2z) \\
 &= \{x+(3y-2z)\}\{x-(3y-2z)\} \\
 &= x^2 - (3y-2z)^2 = x^2 - (9y^2 - 12yz + 4z^2) \\
 &= x^2 - 9y^2 + 12yz - 4z^2
 \end{aligned}$$

$$\begin{aligned}
 (4) & (a+b-c-d)(a-b-c+d) \\
 &= \{(a-c)-(b-d)\}\{(a-c)+(b-d)\} \\
 &= (a-c)^2 - (b-d)^2 \\
 &= a^2 - 2ac + c^2 - b^2 + 2bd - d^2
 \end{aligned}$$

$$\begin{aligned}
 (5) & (a-2b)^2(a+2b)^2(a^2+4b^2)^2 \\
 &= \{(a^2-4b^2)(a^2+4b^2)\}^2 \\
 &= (a^4-16b^4)^2 = a^8 - 32a^4b^4 + 256b^8
 \end{aligned}$$

$$\begin{aligned}
 (6) & (a+b)(a-b)(a^2+b^2)(a^4+b^4) \\
 &= (a^2-b^2)(a^2+b^2)(a^4+b^4) \\
 &= (a^4-b^4)(a^4+b^4) = a^8 - b^8
 \end{aligned}$$

$$\begin{aligned}
 (7) & (a-b)(a+b)(a^2+ab^2+b^2)(a^2-ab^2+b^2) \\
 &= (a^3-b^3)(a^3+b^3) \\
 &= a^6 - b^6
 \end{aligned}$$

$$\begin{aligned}
 (8) & (x-1)(x-3)(x+2)(x+4) \\
 &= (x-1)(x+2) \times (x-3)(x+4) \\
 &= (x^2+x-2)(x^2+x-12) \\
 &= (x^2+x)^2 - 14(x^2+x) + 24 \\
 &= x^4 + 2x^3 + x^2 - 14x^2 - 14x + 24 \\
 &= x^4 + 2x^3 - 13x^2 - 14x + 24
 \end{aligned}$$

数と式 (因数分解)NO3

因数分解の手順 ① 共通因数でくくる $ma + mb = m(a + b)$

1. 次の式を因数分解せよ。

(1) $4(a - 2b) + (a - 2b)y = (a - 2b)(4 + y)$

(2) $a(x - y) + b(y - x) = a(x - y) \mp b(x - y) = (a - b)(x - y)$

(3) $a(2a - 3b) + b(3b - 2a) = a(2a - 3b) \mp b(2a - 3b) = (a - b)(2a - 3b)$

因数分解の手順 ② 公式を適用する

[1] $a^2 + 2ab + b^2 = (a + b)^2$ [2] $a^2 - 2ab + b^2 = (a - b)^2$
 [3] $a^2 - b^2 = (a + b)(a - b)$ [4] $x^2 + (a + b)x + ab = (x + a)(x + b)$

2. 次の式を因数分解せよ。

(1) $x^2 - y^2 + 2y - 1$
 $= x^2 - (y^2 - 2y + 1)$
 $= x^2 - (y - 1)^2$
 $= (x + y - 1)(x - y + 1)$

(2) $a^2 + 9b^2 - 16c^2 - 6ab$
 $= (a^2 - 6ab + 9b^2) - 16c^2$
 $= (a - 3b)^2 - (4c)^2$
 $= (a - 3b + 4c)(a - 3b - 4c)$

[5] $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$

3. 次の式を因数分解せよ。

(1) $6x^2 + x - 2$
 $= (3x + 2)(2x - 1)$

(2) $4x^2 + 8xy - 21y^2$
 $= (2x - 3y)(2x + 7y)$

(3) $3a^2 + 10a + 3$
 $= (3a + 1)(a + 3)$

(4) $3x^2 - 7xy + 2y^2$
 $= (3x - 2y)(x - 2y)$

(5) $3a^2 - 14ab + 8b^2$
 $= (3a - 2b)(a - 4b)$

(6) $x^2 - (a - 1)x - a$
 $= (x + 1)(x - a)$

(7) $ax^2 - (1 + ab)x + b$
 $= (ax - 1)(x - b)$

$$\begin{array}{r} a \quad -1 \quad -1 \\ \times \\ 1 \quad -b \quad -ab \\ \hline \quad \quad -1-ab \end{array}$$

(8) $8x^2 - 51x(y + z) + 18(y + z)^2$
 $= (8x - 3y - 3z)(x - 6y - 6z)$

$$\begin{array}{r} 8 \quad -3(y+z) \quad -3(y+z) \\ \times \\ 1 \quad -6(y+z) \quad -6(y+z) \\ \hline \quad \quad -51(y+z) \end{array}$$

(9) $(a^2 - b^2)x^2 - (a^2 + b^2)x + ab$
 $= (a + b)(a - b)x^2 - (a^2 + b^2)x + ab$
 $= \{(a + b)x - a\} \{(a - b)x - b\}$
 $= (ax + bx - a)(ax - bx - b)$

$$\begin{array}{r} a+b \quad -a \quad -a^2+ab \\ \times \\ a-b \quad -b \quad -ab-b^2 \\ \hline \quad \quad - (a^2+b^2) \end{array}$$

数と式 (因数分解)NO4

[6] $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ [7] $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

4. 次の式を因数分解せよ。

(1) $x^3 + 27$

$= x^3 + 3^3$

$= (x+3)(x^2 - 3x + 9)$

(3) $x^3y^3 - 27z^3$

$= (xy)^3 - (3z)^3$

$= (xy - 3z)(x^2y^2 + 3xy^2z + 9z^2)$

(2) $8a^3 - 27b^3$

$= (2a)^3 - (3b)^3$

$= (2a - 3b)(4a^2 + 6ab + 9b^2)$

(4) $a^3 + (b+1)^3$

$= \{a + (b+1)\} \{a^2 - a(b+1) + (b+1)^2\}$

$= (a+b+1)(a^2 - ab + b^2 - a + 2b + 1)$

因数分解の手順 ③ 最低次数の文字で整理する

例題5 次の式を因数分解せよ。 $a^3 - ab^2 - b^2c + a^2c$
 [解] $a^3 - ab^2 - b^2c + a^2c = (a^2 - b^2)c + (a^3 - ab^2)$
 $= (a^2 - b^2)c + a(a^2 - b^2) = (a^2 - b^2)(c + a) = (a+b)(a-b)(a+c)$

5. 次の式を因数分解せよ。

(1) $x^2 - 9y + 3xy - 9$

$= 3(x-3)y + (x^2 - 9)$

$= 3(x-3)y + (x-3)(x+3)$

$= (x-3)\{3y + (x+3)\}$

$= (x-3)(x+3y+3) \dots$ 答

(1) $x^2 - 9y + 3xy - 9$

(2) $a^2b + a^2 - b - 1$

$= (a^2 - 1)b + (a^2 - 1)$

$= (a^2 - 1)(b + 1)$

$= (a+1)(a-1)(b+1) \dots$ 答

(2) $a^2b + a^2 - b - 1$

(3) $a^2 + b^2 + bc - ca - 2ab$

$= (b-a)c + (a^2 - 2ab + b^2)$

$= -(a-b)c + (a-b)^2$

$= (a-b)\{-c + (a-b)\}$

$= (a-b)(a-b-c) \dots$ 答

(3) $a^2 + b^2 + bc - ca - 2ab$

(4) $x^3 + 3x^2y + zx^2 + 2xy^2 + 3xyz + 2zy^2$

$= (x^2 + 3xy + 2y^2)z + (x^3 + 3x^2y + 2xy^2)$

$= (x^2 + 3xy + 2y^2)z + (x^2 + 3xy + 2y^2)x$

$= (x^2 + 3xy + 2y^2)(z+x)$

$= (x+y)(x+2y)(x+z) \dots$ 答

(4) $x^3 + 3x^2y + zx^2 + 2xy^2 + 3xyz + 2zy^2$

数と式 (因数分解)NO5

6. 次の式を因数分解せよ。

$$\begin{aligned}
 (1) & 2x^2 + 5xy + 2y^2 - 5x - y - 3 \\
 &= 2x^2 + (5y - 5)x + (2y^2 - y - 3) \\
 &= 2x^2 + (5y - 5)x + (y + 1)(2y - 3) \\
 &= (x + 2y - 3)(2x + y + 1) \dots \text{答}
 \end{aligned}$$

$$\begin{array}{r}
 1 \times \begin{array}{r} 2y-3 \\ y+1 \end{array} \quad \begin{array}{r} 4y-6 \\ y+1 \end{array} \\
 \hline
 2 \times \begin{array}{r} y+1 \\ y+1 \end{array} \\
 \hline
 5y-5
 \end{array}$$

$$\begin{aligned}
 (2) & 2x^2 + xy - 3y^2 + 5x + 5y + 2 \\
 &= 2x^2 + (y + 5)x - (3y^2 - 5y - 2) \\
 &= 2x^2 + (y + 5)x - (y - 2)(3y + 1) \\
 &= \{2x + (3y + 1)\}\{x - (y - 2)\} \\
 &= (2x + 3y + 1)(x - y + 2) \dots \text{答}
 \end{aligned}$$

$$\begin{array}{r}
 2 \times \begin{array}{r} 3y+1 \\ -(y-2) \end{array} \quad \begin{array}{r} 3y+1 \\ -2y+4 \end{array} \\
 \hline
 1 \times \begin{array}{r} y+5 \\ y+5 \end{array} \\
 \hline
 y+5
 \end{array}$$

$$\begin{aligned}
 (3) & x^2 - 2xy + y^2 + 4x - 4y + 3 \\
 &= x^2 + (-2y + 4)x + y^2 - 4y + 3 \\
 &= x^2 + (-2y + 4)x + (y - 1)(y - 3) \\
 &= \{x - (y - 1)\}\{x - (y - 3)\} \\
 &= (x - y + 1)(x - y + 3) \dots \text{答}
 \end{aligned}$$

$$\begin{array}{r}
 1 \times \begin{array}{r} -(y-1) \\ -(y-3) \end{array} \quad \begin{array}{r} -y+1 \\ -y+3 \end{array} \\
 \hline
 1 \times \begin{array}{r} -2y+4 \\ -2y+4 \end{array} \\
 \hline
 -2y+4
 \end{array}$$

$$\begin{aligned}
 (4) & 6x^2 + 5xy + x - 6y^2 - 5y - 1 \\
 &= 6x^2 + (5y + 1)x - (6y^2 + 5y + 1) \\
 &= 6x^2 + (5y + 1)x - (3y + 1)(2y + 1) \\
 &= \{2x + (3y + 1)\}\{3x - (2y + 1)\} \\
 &= (2x + 3y + 1)(3x - 2y - 1) \dots \text{答}
 \end{aligned}$$

$$\begin{array}{r}
 2 \times \begin{array}{r} 3y+1 \\ -(2y+1) \end{array} \quad \begin{array}{r} 9y+3 \\ -4y-2 \end{array} \\
 \hline
 3 \times \begin{array}{r} (2y+1) \\ -(2y+1) \end{array} \\
 \hline
 5y+1
 \end{array}$$

7. 次の式を因数分解せよ。

$$\begin{aligned}
 (1) & x^3 + (a + 2)x^2 + (2a + 1)x + a \\
 &= x^3 + ax^2 + 2x^2 + 2ax + 2x + a \\
 &= (x^2 + 2x + 1)a + x^3 + 2x^2 + 2x \\
 &= (x^2 + 2x + 1)a + x(x^2 + 2x + 1) \\
 &= (x^2 + 2x + 1)(a + x) \\
 &= (x + 1)^2(x + a)
 \end{aligned}$$

$$(1) 2x^2 + 5xy + 2y^2 - 5x - y - 3$$

$$(2) 2x^2 + xy - 3y^2 + 5x + 5y + 2$$

$$(3) x^2 - 2xy + y^2 + 4x - 4y + 3$$

$$(4) 6x^2 + 5xy + x - 6y^2 - 5y - 1$$

$$\begin{aligned}
 (2) & x^2 + x - y^2 + 5y - 6 \\
 &= x^2 + x - (y^2 - 5y + 6) \\
 &= x^2 + x - (y - 2)(y - 3) \\
 &= (x + y - 2)\{x - (y - 3)\} \\
 &= \underline{(x + y - 2)(x - y + 3)}
 \end{aligned}$$

$$\begin{array}{r}
 1 \times \begin{array}{r} y-2 \\ -(y-3) \end{array} \quad \begin{array}{r} y-2 \\ -y+3 \end{array} \\
 \hline
 1 \times \begin{array}{r} x \\ x \end{array} \\
 \hline
 x
 \end{array}$$

数と式 (因数分解)NO6

8. 次の式を因数分解せよ

$$\begin{aligned}
 (1) & a^2(b-c) + b^2(c-a) + c^2(a-b) \\
 &= \boxed{a^2(b-c) + b^2c - ab^2 + ac^2 - bc^2} \\
 &= (b-c)a^2 - (b^2 - c^2)a + (b^2c - bc^2) \\
 &= (b-c)a^2 - (b-c)(b+c)a \\
 &\qquad\qquad\qquad + bc(b-c) \\
 &= (b-c)\{a^2 - (b+c)a + bc\} \\
 &= (b-c)(a-b)(a-c) \\
 &= -(a-b)(b-c)(c-a) \dots \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 (2) & a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) \\
 &= \boxed{ab^2 - ac^2 + bc^2 - a^2b + a^2c - b^2c} \\
 &= (c-b)a^2 - (c^2 - b^2)a + (bc^2 - b^2c) \\
 &= (c-b)a^2 - (c-b)(c+b)a \\
 &\qquad\qquad\qquad + bc(c-b) \\
 &= (c-b)\{a^2 - (c+b)a + bc\} \\
 &= (c-b)(a-c)(a-b) \\
 &= (a-b)(b-c)(c-a) \dots \text{答}
 \end{aligned}$$

$$\begin{aligned}
 (3) & a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc \\
 &= \boxed{ab^2 + ac^2 + bc^2 + a^2b + a^2c + b^2c + 2abc} \\
 &= (b+c)a^2 + (b^2 + 2bc + c^2)a + (b^2c + bc^2) \\
 &= (b+c)a^2 + (b+c)^2a + bc(b+c) \\
 &= (b+c)\{a^2 + (b+c)a + bc\} \\
 &= (b+c)(a+b)(a+c) \\
 &= (a+b)(b+c)(c+a) \dots \text{答}
 \end{aligned}$$

$$\begin{aligned}
 (4) & (a+b)(b+c)(c+a) + abc \\
 &= (a+b)(bc + ab + c^2 + ac) + abc \\
 &= (a+b)\{(a+b)c + (ab + c^2)\} + abc \\
 &\quad a+b = A \text{ とおくと,} \\
 (\text{与式}) &= A\{Ac + (ab + c^2)\} + abc \\
 &= A^2c + (ab + c^2)A + abc \\
 &= (A+c)(Ac + ab) \quad \begin{array}{r} \times \begin{array}{cc} c & c^2 \\ ab & ab \end{array} \\ \hline ab + c^2 \end{array} \\
 &= (a+b+c)\{(a+b)c + ab\} \\
 &= (a+b+c)(ab + bc + ca) \dots \text{答}
 \end{aligned}$$

$$(1) a^2(b-c) + b^2(c-a) + c^2(a-b)$$

$$(2) a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$$

$$(3) a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$$

$$(4) (a+b)(b+c)(c+a) + abc$$

数と式 (因数分解)NO7

9. 次の式を因数分解せよ

$$\begin{aligned}
 (5) & (a+b+c)(ab+bc+ca) - abc \\
 &= \{a+(b+c)\} \{(b+c)a+bc\} - abc \\
 & \quad b+c=A \text{ とおくと} \\
 (\text{与式}) &= (a+A)(Aa+bc) - abc \\
 &= Aa^2 + A^2a + Abc \\
 &= A(a^2 + Aa + bc) \\
 &= (b+c)\{a^2 + (b+c)a + bc\} \\
 &= (b+c)(a+b)(a+c) \\
 &= \underline{(a+b)(b+c)(c+a)}
 \end{aligned}$$

$$\begin{aligned}
 (6) & (a+b+c+1)(a+1) + bc \\
 &= \{(a+1) + (b+c)\}(a+1) + bc \\
 & \quad a+1=A \text{ とおくと} \\
 (\text{与式}) &= \{A+(b+c)\}A + bc \\
 &= A^2 + (b+c)A + bc \\
 &= (A+b)(A+c) \\
 &= \underline{(a+b+1)(a+c+1)}
 \end{aligned}$$

$$(5) (a+b+c)(ab+bc+ca) - abc$$

$$(6) (a+b+c+1)(a+1) + bc$$

10. 次の式を因数分解せよ

$$\begin{aligned}
 (1) & (x-3)(x-1)(x+3)(x+5) + 35 \\
 &= (x-3)(x+5) \times (x-1)(x+3) + 35 \\
 &= (x^2+2x-15)(x^2+2x-3) + 35 \\
 & \quad x^2+2x=A \text{ とおくと} \\
 (\text{与式}) &= (A-15)(A-3) + 35 \\
 &= A^2 - 18A + 80 \\
 &= (A-8)(A-10) \\
 &= (x^2+2x-8)(x^2+2x-10) \\
 &= \underline{(x-2)(x+4)(x^2+2x-10)}
 \end{aligned}$$

$$\begin{aligned}
 (2) & (x-1)(x-2)(x-3)(x-6) - 3x^2 \\
 &= (x-1)(x-6) \times (x-2)(x-3) \\
 &= (x^2-7x+6)(x^2-5x+6) - 3x^2 \\
 & \quad x^2+6=A \text{ とおくと} \\
 (\text{与式}) &= (A-7x)(A-5x) - 3x^2 \\
 &= A^2 - 12xA + 32x^2 \\
 &= (A-4x)(A-8x) \\
 &= \underline{(x^2-4x+6)(x^2-8x+6)}
 \end{aligned}$$

$$(1) (x-3)(x-1)(x+3)(x+5) + 35$$

$$(2) (x-1)(x-2)(x-3)(x-6) - 3x^2$$

数と式 (因数分解)NO8

11. 次の式を因数分解せよ

$$\begin{aligned} (1) x^4 - 5x^2 + 4 \\ &= (x^2 - 1)(x^2 - 4) \\ &= \underline{(x+1)(x-1)(x+2)(x-2)} \end{aligned}$$

$$\begin{aligned} (2) x^4 - 10x^2 + 9 \\ &= (x^2 - 1)(x^2 - 9) \\ &= \underline{(x+1)(x-1)(x+3)(x-3)} \end{aligned}$$

$$\begin{aligned} (3) 4x^4 - 37x^2y^2 + 9y^4 \\ &= (4x^2 - y^2)(x^2 - 9y^2) \\ &= \underline{(2x+y)(2x-y)(x+3y)(x-3y)} \end{aligned}$$

$$\begin{aligned} (4) x^6 - 7x^3 - 8 \\ &= (x^3 + 1)(x^3 - 8) \\ &= (x+1)(x^2 - x + 1)(x-2)(x^2 + 2x + 4) \\ &= \underline{(x+1)(x-2)(x^3 - x + 1)(x^2 + 2x + 4)} \end{aligned}$$

$$\begin{aligned} (5) x^4 + 4 \\ &= (x^2 + 2)^2 - 4x^2 \\ &= (x^2 + 2)^2 - (2x)^2 \\ &= \underline{(x^2 + 2x + 2)(x^2 - 2x + 2)} \end{aligned}$$

$$\begin{aligned} (6) x^4 + 5x^2 + 9 \\ &= (x^2 + 3)^2 - x^2 \\ &= \underline{(x^2 + x + 3)(x^2 - x + 3)} \end{aligned}$$

$$\begin{aligned} (7) x^4 - 7x^2y^2 + y^4 \\ &= (x^2 + y^2)^2 - 9x^2y^2 \\ &= (x^2 + y^2)^2 - (3xy)^2 \\ &= \underline{(x^2 + 3xy + y^2)(x^2 - 3xy + y^2)} \end{aligned}$$

$$\begin{aligned} (8) x^4 - 8x^2 + 4 \\ &= (x^2 - 2)^2 - 4x^2 \\ &= (x^2 - 2)^2 - (2x)^2 \\ &= \underline{(x^2 + 2x - 2)(x^2 - 2x - 2)} \end{aligned}$$

$$(1) x^4 - 5x^2 + 4$$

$$(2) x^4 - 10x^2 + 9$$

$$(3) 4x^4 - 37x^2y^2 + 9y^4$$

$$(4) x^6 - 7x^3 - 8$$

$$(5) x^4 + 4$$

$$(6) x^4 + 5x^2 + 9$$

$$(7) x^4 - 7x^2y^2 + y^4$$

$$(8) x^4 - 8x^2 + 4$$

数と式 (因数分解) NO9

12. 次の式を因数分解せよ

(1) $a^3 + b^3 + c^3 - 3abc$

$= \underline{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}$

(2) $8x^3 - y^3 - 6xy - 1$

$= (2x)^3 + (-y)^3 + (-1)^3 - 3(2x)(-y)(-1)$

$= (2x-y-1)(4x^2+y^2+1+2x-y+2x)$

$= \underline{(2x-y-1)(4x^2+2x-y+y^2+2x-y+1)}$

(3) $(a-b)^3 + (b-c)^3 + (c-a)^3$

$a-b=A, b-c=B, c-a=C$ とおくと

$A+B+C=0$

$A^3+B^3+C^3-3ABC$

$= (A+B+C)(A^2+B^2+C^2-AB-BC-CA) = 0$

$\therefore A^3+B^3+C^3 = 3ABC$

したがって

$(a-b)^3 + (b-c)^3 + (c-a)^3 = \underline{3(a-b)(b-c)(c-a)}$

(4) $(a-x)^3 + (b-x)^3 - (a+b-2x)^3$

$a-x=X, b-x=Y$ とおくと

$a+b-2x = X+Y$

(与式) $= X^3 + Y^3 - (X+Y)^3$

$= -3XY(X+Y)$

$= -3(a-x)(b-x)(a+b-2x)$

$= 3(a-x)(b-x)(2x-a-b)$

(1) $a^3 + b^3 + c^3 - 3abc$

(2) $8x^3 - y^3 - 6xy - 1$

(3) $(a-b)^3 + (b-c)^3 + (c-a)^3$

(4) $(a-x)^3 + (b-x)^3 - (a+b-2x)^3$

(5) $a^3(b-c) + b^3(c-a) + c^3(a-b)$

$= (b-c)a^3 + b^3c - ab^3 + ac^3 - bc^3$

$= (b-c)a^3 - (b^3-c^3)a + b^3c - bc^3$

$= (b-c)a^3 - (b-c)(b^2+bc+c^2)a + b^3c - bc^3$

$= (b-c)\{a^3 - (b^2+bc+c^2)a + bc(b+c)\}$

$= (b-c)\{a^3 - ab^2 - abc - ac^2 + b^2c + bc^2\}$

$= (b-c)\{c(c-a)b^2 + (c^2-ac)b + (a^3-ac^2)\}$

$= (b-c)\{c(c-a)b^2 + c(c-a)b - a(c^2-a^2)\}$

$= (b-c)(c-a)\{b^2+bc-a(c+a)\}$

$= (b-c)(c-a)(b^2+bc-ac-a^2)$

$= (b-c)(c-a)\{(b-a)c + (b^2-a^2)\}$

$= (b-c)(c-a)(b-a)(c+b+a)$

$= \underline{-(a-b)(b-c)(c-a)(a+b+c)}$