

数と式 (展開)NO1

1. 次の式を x について降べきの順に整理せよ。

$$x + 2y^2 + 3x^2 - 5y + 3 + 2xy$$

2. 次の式を展開せよ。

乗法公式

$$[6](a + b)^3 = \boxed{}$$

$$[7](a - b)^3 = \boxed{}$$

$$[8](a + b)(a^2 - ab + b^2) = \boxed{}$$

$$[9](a - b)(a^2 + ab + b^2) = \boxed{}$$

$$(1) (2x + 3)^3 = (\quad)^{\square} + 3 \cdot (\quad)^{\square} \cdot \boxed{} + 3 \cdot (\quad) \cdot \boxed{}^{\square} + \boxed{}^{\square} = \boxed{}$$

$$(2) (3a - 2b)^3 = (\quad)^{\square} \boxed{} 3 \cdot (\quad)^{\square} \cdot (\quad) + 3 \cdot (\quad) \cdot (\quad)^{\square} \boxed{} (\quad)^{\square} = \boxed{}$$

$$(3) (2a + b)(4a^2 - 2ab + b^2) = (2a + b)\{(\quad)^{\square} - (\quad) \cdot \boxed{} + b^2\} = \boxed{}$$

$$(4) (3x - 4y)(9x^2 + 12xy + 16y^2) = (3x - 4y)\{(\quad)^{\square} + (\quad) \cdot (\quad) + (\quad)^{\square}\} = \boxed{}$$

置き換えによる展開の工夫

3. 次の式を展開せよ。

$$(1) (a + b - 1)(a + b + 3)$$

$a + b = A$ とおくと

$$(a + b - 1)(a + b + 3) = (A - 1)(A + 3)$$

$$= A^2 + 2A - 3 = (a + b)^2 + 2(a + b) - 3$$

$$= a^2 + 2ab + b^2 + 2a + 2b - 3$$

$$(2) (a + b - c)(a - b + c) = \{a + (b - c)\}\{a - (b - c)\}$$

$b - c = A$ とおくと

$$(a + b - c)(a - b + c) = (a + A)(a - A)$$

$$= a^2 - A^2 = a^2 - (b - c)^2$$

$$= a^2 - (b^2 - 2bc + c^2) = a^2 - b^2 + 2bc - c^2$$

$$(a + b + c)^2 = \boxed{}$$

4. 次の式を展開せよ。

$$(1) (2x + y - z)^2 = 4x^2 + y^2 + z^2 + 2 \cdot (2x) \cdot \boxed{} + 2 \cdot y \cdot (-z) + \boxed{} \cdot (-z) \cdot (\quad)$$

$$= \boxed{}$$

応用問題

$$(1) (x + 1)(x + 2)(x + 3)(x + 4)$$

$$= (x + 1)(x + 4) \times (x + 2)(x + 3)$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6)$$

$x^2 + 5x = A$ とおくと、

$$(x + 1)(x + 2)(x + 3)(x + 4) = (A + 4)(A + 6)$$

$$= A^2 + 10A + 24 = (x^2 + 5x)^2 + 10(x^2 + 5x) + 24$$

$$= x^4 + 10x^3 + 25x^2 + 10x^2 + 50x + 24$$

$$= x^4 + 10x^3 + 35x^2 + 50x + 24$$

$$(2) (x + 1)(x + 2)(x + 3)(x + 6)$$

$$= (x + 1)(x + 6) \times (x + 2)(x + 3)$$

$$= (x^2 + 7x + 6)(x^2 + 5x + 6)$$

$x^2 + 6 = A$ とおくと、

$$(x + 1)(x + 2)(x + 3)(x + 6) = (A + 7x)(A + 5x)$$

$$= A^2 + 12Ax + 35x^2$$

$$= (x^2 + 6)^2 + 12(x^2 + 6)x + 35x^2$$

$$= x^4 + 12x^3 + 36 + 12x^3 + 72x + 35x^2$$

$$= x^4 + 12x^3 + 47x^2 + 72x + 36$$

数と式 (展開)NO2

$$\begin{aligned}
 (3) & (a-b)(a+b)(a^4+a^2b^2+b^4) \\
 &= (a^2-b^2)(a^4+a^2b^2+b^4) \\
 &= (a^2)^3 - (b^2)^3 \\
 &= a^6 - b^6
 \end{aligned}$$

$$\begin{aligned}
 (4) & (a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2) \\
 &= (a-b)(a^2+ab+b^2) \times (a+b)(a^2-ab+b^2) \\
 &= (a \square - b \square)(a \square + b \square) \\
 &= a \square - b \square
 \end{aligned}$$

$$\begin{aligned}
 (5) & (2x+3)^2(2x-3)^2 \\
 &= \{(2x+3)(2x-3)\}^2 \\
 &= (4x^2-9)^2 \\
 &= 16x^4 - 72x^2 + 81
 \end{aligned}$$

$$\begin{aligned}
 (6) & (x-2)^3(x+2)^3 \\
 &= \{(x-2)(x+2)\}^3 \\
 &= (x^2-4)^3 \\
 &=
 \end{aligned}$$

$$\begin{aligned}
 (7) & (a-b)(a+b)(a^2+b^2)(a^4+b^4) \\
 &= (a^2-b^2)(a^2+b^2)(a^4+b^4) \\
 &= (a^4-b^4)(a^4+b^4) \\
 &= a^8 - b^8
 \end{aligned}$$

$$\begin{aligned}
 (8) & (a-b)^3(a+b)^3(a^2+b^2)^3 \\
 &= \{(a-b)(a+b)(a^2+b^2)\}^3 \\
 &= \{(a^2-b^2)(a^2+b^2)\}^3 = (a^4-b^4)^3 \\
 &=
 \end{aligned}$$

5. 次の式を展開せよ。

(1) $(3a-2b)^3$

(2) $(2a-3b-c)^2$

(3) $(x+3y-2z)(x-3y+2z)$

(4) $(a+b-c-d)(a-b-c+d)$

(5) $(a-2b)^2(a+2b)^2(a^2+4b^2)^2$

(6) $(a+b)(a-b)(a^2+b^2)(a^4+b^4)$

(7) $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$

(8) $(x-1)(x-3)(x+2)(x+4)$
 $= (x-1)(\quad) \times (x-3)(\quad)$

数と式 (因数分解)NO3

因数分解の手順 共通因数でくくる $ma + mb = m(a + b)$

1. 次の式を因数分解せよ。

(1) $4(a - 2b) + (a - 2b)y = (\quad) (4 + y)$

(2) $a(x - y) + b(y - x) = a(x - y) \square b(x - y) = (\quad) (x - y)$

(3) $a(2a - 3b) + b(3b - 2a) = a(2a - 3b) \square b(2a - 3b) = (\quad) (\quad)$

因数分解の手順 公式を適用する

[1] $a^2 + 2ab + b^2 = \square$

[2] $a^2 - 2ab + b^2 = \square$

[3] $a^2 - b^2 = (\quad) (\quad)$

[4] $x^2 + (a + b)x + ab = \square$

2. 次の式を因数分解せよ。

(1) $x^2 - y^2 + 2y - 1$

(2) $a^2 + 9b^2 - 16c^2 - 6ab$

[5] $acx^2 + (ad + bc)x + bd = \square$

3. 次の式を因数分解せよ。

(1) $6x^2 + x - 2$

(2) $4x^2 + 8xy - 21y^2$

(3) $3a^2 + 10a + 3$

(4) $3x^2 - 7xy + 2y^2$

(5) $3a^2 - 14ab + 8b^2$

(6) $x^2 - (a - 1)x - a$

(7) $ax^2 - (1 + ab)x + b$

(8) $8x^2 - 51x(y + z) + 18(y + z)^2$

(9) $(a^2 - b^2)x^2 - (a^2 + b^2)x + ab$

数と式 (因数分解)NO4

[6] $a^3 + b^3 = (\quad) (\quad)$ [7] $a^3 - b^3 = (\quad) (\quad)$

4. 次の式を因数分解せよ。

(1) $x^3 + 27$

(2) $8a^3 - 27b^3$

(3) $x^3y^3 - 27z^3$

(4) $a^3 + (b + 1)^3$

因数分解の手順 最低次数の文字で整理する

例題5 次の式を因数分解せよ。 $a^3 - ab^2 - b^2c + a^2c$
 [解] $a^3 - ab^2 - b^2c + a^2c = (\quad)c + (\quad)$
 $= (\quad)c + a(\quad) = (\quad) (\quad) = (\quad) (\quad) (\quad)$

5. 次の式を因数分解せよ。

(1) $x^2 - 9y + 3xy - 9$
 $= 3(x - 3)y + (\quad)$
 $= 3(x - 3)y + (\quad) (\quad)$
 $= (x - 3)\{3y + (\quad)\}$
 $= (x - 3)(\quad) \dots$ 答

(1) $x^2 - 9y + 3xy - 9$

(2) $a^2b + a^2 - b - 1$
 $= (a^2 - 1)b + (\quad)$
 $= (\quad)(b + 1)$
 $= (\quad)(\quad)(\quad) \dots$ 答

(2) $a^2b + a^2 - b - 1$

(3) $a^2 + b^2 + bc - ca - 2ab$
 $= (\quad)c + (a^2 - 2ab + b^2)$
 $= -(\quad)c + (a - b)^2$
 $= (\quad)\{-c + (\quad)\}$
 $= (\quad)(\quad) \dots$ 答

(3) $a^2 + b^2 + bc - ca - 2ab$

(4) $x^3 + 3x^2y + zx^2 + 2xy^2 + 3xyz + 2zy^2$
 $= (\quad)z + (x^3 + 3x^2y + 2xy^2)$
 $= (\quad)z + (x^2 + 3xy + 2y^2)x$
 $= (\quad)(\quad)$
 $= (\quad)(\quad)(\quad) \dots$ 答

(4) $x^3 + 3x^2y + zx^2 + 2xy^2 + 3xyz + 2zy^2$

数と式 (因数分解)NO5

6. 次の式を因数分解せよ。

$$\begin{aligned}
 (1) & 2x^2 + 5xy + 2y^2 - 5x - y - 3 \\
 &= 2x^2 + (\quad) x + (2y^2 - y - 3) \\
 &= 2x^2 + (\quad) x + (\quad)(\quad) \\
 &= (\quad)(\quad) \cdots \text{答}
 \end{aligned}$$

$$(1) 2x^2 + 5xy + 2y^2 - 5x - y - 3$$

$$\begin{aligned}
 (2) & 2x^2 + xy - 3y^2 + 5x + 5y + 2 \\
 &= 2x^2 + (\quad) x - (\quad) \\
 &= 2x^2 + (\quad) x - (\quad)(\quad) \\
 &= \{2x + (\quad)\} \{x - (\quad)\} \\
 &= (\quad)(\quad) \cdots \text{答}
 \end{aligned}$$

$$(2) 2x^2 + xy - 3y^2 + 5x + 5y + 2$$

$$\begin{aligned}
 (3) & x^2 - 2xy + y^2 + 4x - 4y + 3 \\
 &= x^2 + (-2y + 4)x + y^2 - 4y + 3 \\
 &= x^2 + (\quad) x + (\quad)(\quad) \\
 &= \{x - (\quad)\} \{x - (\quad)\} \\
 &= (\quad)(\quad) \cdots \text{答}
 \end{aligned}$$

$$(3) x^2 - 2xy + y^2 + 4x - 4y + 3$$

$$\begin{aligned}
 (4) & 6x^2 + 5xy + x - 6y^2 - 5y - 1 \\
 &= 6x^2 + (\quad) x - (6y^2 + 5y + 1) \\
 &= 6x^2 + (\quad) x - (\quad)(\quad) \\
 &= \{2x + (\quad)\} \{3x - (\quad)\} \\
 &= (\quad)(\quad) \cdots \text{答}
 \end{aligned}$$

$$(4) 6x^2 + 5xy + x - 6y^2 - 5y - 1$$

7. 次の式を因数分解せよ。

$$(1) x^3 + (a+2)x^2 + (2a+1)x + a$$

$$(2) x^2 + x - y^2 + 5y - 6$$

数と式 (因数分解)NO6

8. 次の式を因数分解せよ

$$\begin{aligned}
 (1) & a^2(b-c) + b^2(c-a) + c^2(a-b) \\
 &= \boxed{} \\
 &= ()a^2 - ()a + () \\
 &= ()a^2 - ()()a \\
 &\qquad\qquad\qquad + bc() \\
 &= ()\{a^2 - ()a + bc\} \\
 &= ()()() \\
 &= -()()() \dots \text{答}
 \end{aligned}$$

$$(1) a^2(b-c) + b^2(c-a) + c^2(a-b)$$

$$\begin{aligned}
 (2) & a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) \\
 &= \boxed{} \\
 &= (c-b)a^2 - ()a + (bc^2 - b^2c) \\
 &= (c-b)a^2 - ()()a \\
 &\qquad\qquad\qquad + bc() \\
 &= (c-b)\{a^2 - ()a + bc\} \\
 &= ()()() \\
 &= ()()() \dots \text{答}
 \end{aligned}$$

$$(2) a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$$

$$\begin{aligned}
 (3) & a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc \\
 &= \boxed{} \\
 &= ()a^2 + ()a + () \\
 &= ()a^2 + ()^2a + bc() \\
 &= ()\{a^2 + ()a + bc\} \\
 &= ()()() \\
 &= ()()() \dots \text{答}
 \end{aligned}$$

$$(3) a(b-c)^2 + b(c-a)^2 + c(a-b)^2 + 8abc$$

$$\begin{aligned}
 (4) & (a+b)(b+c)(c+a) + abc \\
 &= (a+b)(bc + ab + c^2 + ac) + abc \\
 &= (a+b)\{()c + (ab + c^2)\} + abc \\
 &\quad a + b = A \text{ とおくと,} \\
 \text{(与式)} &= A\{Ac + ()\} + abc \\
 &= A^2c + ()A + abc \\
 &= (A+c)() \\
 &= ()\{(a+b)c + ab\} \\
 &= ()() \dots \text{答}
 \end{aligned}$$

$$(4) (a+b)(b+c)(c+a) + abc$$

数と式 (因数分解)NO7

9. 次の式を因数分解せよ

$$(5) (a + b + c)(ab + bc + ca) - abc$$
$$= \{a + (b + c)\} \{(\quad) a + bc\} - abc$$

$$b + c = A \text{ とおくと}$$

$$\text{(与式)} =$$

$$(5) (a + b + c)(ab + bc + ca) - abc$$

$$(6) (a + b + c + 1)(a + 1) + bc$$
$$= \{(a + 1) + (b + c)\}(a + 1) + bc$$

$$a + 1 = A \text{ とおくと}$$

$$\text{(与式)} =$$

$$(6) (a + b + c + 1)(a + 1) + bc$$

10. 次の式を因数分解せよ

$$(1) (x - 3)(x - 1)(x + 3)(x + 5) + 35$$
$$= (x - 3)(\quad) \times (x - 1)(\quad) + 35$$
$$= (x^2 + 2x - 15)(x^2 + 2x - 3) + 35$$

$$x^2 + 2x = A \text{ とおくと}$$

$$\text{(与式)} = (A - 15)(A - 3) + 35$$

$$(1) (x - 3)(x - 1)(x + 3)(x + 5) + 35$$

$$(2) (x - 1)(x - 2)(x - 3)(x - 6) - 3x^2$$
$$= (x - 1)(\quad) \times (x - 2)(\quad)$$
$$= (x^2 - 7x + 6)(x^2 - 5x + 6) - 3x^2$$

$$x^2 + 6 = A \text{ とおくと}$$

$$\text{(与式)} = (A - 7x)(A - 5x) - 3x^2$$

=

$$(2) (x - 1)(x - 2)(x - 3)(x - 6) - 3x^2$$

数と式 (因数分解)NO8

11. 次の式を因数分解せよ

$$\begin{aligned} (1) x^4 - 5x^2 + 4 \\ &= (\quad) (\quad) \\ &= (\quad) (\quad) (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (2) x^4 - 10x^2 + 9 \\ &= (\quad) (\quad) \\ &= (\quad) (\quad) (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (3) 4x^4 - 37x^2y^2 + 9y^4 \\ &= (4x^2 - y^2)(\quad) \\ &= (\quad) (\quad) (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (4) x^6 - 7x^3 - 8 \\ &= (\quad) (\quad) \\ &= (\quad) (\quad) (\quad) (\quad) \\ &= (\quad) (\quad) (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (5) x^4 + 4 \\ &= (\quad)^2 - 4x^2 \\ &= (\quad)^2 - (\quad)^2 \\ &= (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (6) x^4 + 5x^2 + 9 \\ &= (\quad)^2 - x^2 \\ &= (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (7) x^4 - 7x^2y^2 + y^4 \\ &= (\quad)^2 - 9x^2y^2 \\ &= (\quad)^2 - (\quad)^2 \\ &= (\quad) (\quad) \end{aligned}$$

$$\begin{aligned} (8) x^4 - 8x^2 + 4 \\ &= (\quad)^2 - 4x^2 \\ &= (\quad)^2 - (\quad)^2 \\ &= (\quad) (\quad) \end{aligned}$$

$$(1) x^4 - 5x^2 + 4$$

$$(2) x^4 - 10x^2 + 9$$

$$(3) 4x^4 - 37x^2y^2 + 9y^4$$

$$(4) x^6 - 7x^3 - 8$$

$$(5) x^4 + 4$$

$$(6) x^4 + 5x^2 + 9$$

$$(7) x^4 - 7x^2y^2 + y^4$$

$$(8) x^4 - 8x^2 + 4$$

数と式 (因数分解)NO9

12. 次の式を因数分解せよ

$$(1) a^3 + b^3 + c^3 - 3abc \\ = (a + b + c)(\quad)$$

$$(1) a^3 + b^3 + c^3 - 3abc$$

$$(2) 8x^3 - y^3 - 6xy - 1 \\ = (2x)^3 + (-y)^3 + (-1)^3 - 3(2x)(-y)(-1) \\ =$$

$$(2) 8x^3 - y^3 - 6xy - 1$$

$$(3) (a - b)^3 + (b - c)^3 + (c - a)^3 \\ a - b = A, b - c = B, c - a = C \text{ とおくと} \\ A + B + C = 0 \\ A^3 + B^3 + C^3 - 3ABC \\ =$$

$$(3) (a - b)^3 + (b - c)^3 + (c - a)^3$$

$$(4) (a - x)^3 + (b - x)^3 - (a + b - 2x)^3 \\ a - x = X, b - x = Y \text{ とおくと} \\ a + b - 2x = X + Y \\ (\text{与式}) = X^3 + Y^3 - (X + Y)^3 \\ = -3XY(X + Y) \\ = -3(a - x)(b - x)(a + b - 2x) \\ = 3(a - x)(b - x)(2x - a - b)$$

$$(4) (a - x)^3 + (b - x)^3 - (a + b - 2x)^3$$

$$(5) a^3(b - c) + b^3(c - a) + c^3(a - b)$$